Teaching ideas for Topic 6: *Atomic and nuclear physics*, AHL

Questions

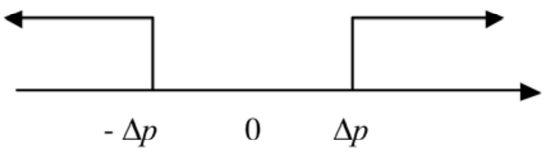
A number of worksheets are provided for this Topic:

- support questions examine the very basic concepts of the syllabus
- extended questions delve deeper and are equivalent to exam level questions.

Teaching ideas

This is a topic with some abstract ideas, so it is worthwhile making it as concrete as possible.

- Many students think that if the electrons leaving a metal surface are faster then this will lead to larger currents. It should be stressed that this is not true – the current only depends on the *number* of electrons being ejected from the surface per second.
- Intensity of radiation, I, means power per unit area so $I = \frac{P}{A}$. Then $P = Nhf$ where N is the number of photons incident per unit time and so $I = \frac{PNhf}{A} = \Phi hf$ where Φ is the number of photons per unit time per unit area. So if the frequency of photons is increased and the intensity stays the same, Φ will have to be decreased, leading to less current from the photosurface.
- It is interesting to tell students that closest distance experiments that determine nuclear radii conclude that $R \approx R_0 A^{1/3}$ and so all nuclei have the same density since $\rho = \frac{M}{V} \approx \frac{A}{(A^{1/3})^3} \sim \text{constant}$.
- Order of magnitude uses of the uncertainty principle reach interesting conclusions such as, for example, that an electron cannot exist within the nucleus.
- It is interesting to point out to students that the energy of an electron in the box model is of the same order of magnitude as that predicted by use of the uncertainty principle: $\Delta x \Delta p \approx h$ (do not worry about the 4π for a rough estimate) and so $\Delta p \approx \frac{h}{\Delta x} \approx \frac{h}{L}$ and so $E_K \approx \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m} \approx \frac{h^2}{2mL^2}$. (Some people object to using $p^2 \approx \Delta p^2$, so see the next bullet point.)
- The previous bullet point involves a calculation whose logic is the following: an uncertainty in position Δx implies an uncertainty in the momentum, $\Delta p \approx \frac{h}{4\pi \Delta x}$. Now the momentum will be measured to be $p_0 \pm \Delta p$. The least magnitude of p_0 is 0 and so the least possible magnitude of the momentum of the electron is Δp. The energy of the electron is then *at least* $E_K \approx \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m}$.



- It is interesting to point out to students that the uncertainty principle states that the electron in the electron in the box model cannot be at rest, just as the energy levels of the box model also require.

- The idea of a wavefunction can be made much more accessible if we realise that the quantity $|\Psi|^2$ is a probability distribution function. This is something that both higher-level and standard-level maths students will have been exposed to, so this is a familiar concept. Then $|\Psi|^2\,\delta x$ is the probability for finding the electron within a distance δx of the point with coordinate x. Since this is on their maths syllabus you can even give a normalized wavefunction, such as $\Psi(x)=\sqrt{\frac{2}{L}}\sin\frac{{}^{1}x}{L}$ and then ask for the probability of, say,

$$P(\frac{L}{3}\le x\le\frac{L}{2})\ :\ P(\frac{L}{3}\le x\le\frac{L}{2})=\frac{2}{L}\int_{\frac{L}{3}}^{\frac{L}{2}}\sin^2\frac{{}^{1}x}{L}\,\mathrm{d}x=0.304\ .$$

- In beta decay, $n^0\to p^+ + e^- + \bar{\nu}$, many students believe that the electron is somehow emitted from within the neutron, i.e. that the neutron turns itself into a proton by emitting the electron that it contains. That this *cannot* be the case is an excellent application of the uncertainty principle, as well as of relativistic mechanics. The following is a demonstration that the electron cannot be inside a neutron. This requires some relativity.
  Assume that the electron is somehow confined within the neutron. The diameter of the neutron is of order 10^{-15} m and so the position of an electron within the neutron has an uncertainty of ½ × 10^{-15} m or simply $\Delta x\approx 10^{-15}$ m as an order of magnitude. Then, by the uncertainty principle, $\Delta p>\frac{h}{4\pi\Delta x}$. This evaluates to $\Delta p>\frac{6.6\times10^{-34}}{4\pi\times10^{-15}}\approx 5\times10^{-20}$ N s. This is best expressed in relativistic units: $\Delta p>\frac{6.6\times10^{-34}}{4\pi\times10^{-15}\times1.6\times10^{-19}}\times\frac{3\times10^{8}}{c}\approx 98\,\frac{\text{MeV}}{c}$. The momentum of the electron is therefore at least $98\,\frac{\text{MeV}}{c}$. From the relativistic formula for energy we find a total energy of at least $E=\sqrt{0.511^2+98^2}\approx 98$ MeV. Now, the binding energy of a small nucleus is less than 100 MeV and so an electron with energy 98 MeV inside the nucleus would rip the nucleus apart – the electron could not have existed within the nucleus.
- Students may be amused to know that Schrödinger had no idea of what the wavefunction was supposed to represent. It was Max Born who gave the wavefunction the probabilistic interpretation it has today.
- The Michael Frayn 1998 play *Copenhagen* (about a meeting in German-occupied Copenhagen in 1941 between Niels Bohr and Werner Heisenberg) is worth seeing if staged anywhere near where you are.
  It can be seen more easily in its 2002 TV film version directed by Howard Davies. Available from Image Entertainment, http://www.image-entertainment.com/

Practical activities/ICT

- A classic simulation of the photoelectric effect is provided at http://phet.colorado.edu/en/simulation/photoelectric
- A simulation about various aspects of the hydrogen atoms is http://phet.colorado.edu/en/simulation/hydrogen-atom
- A simulation with wave properties of particles is http://phet.colorado.edu/en/simulation/davisson-germer
- A very simple simulation of beta decay is http://phet.colorado.edu/en/simulation/beta-decay

Common problems

- Students often get confused between formulae for photons and de Broglie formulae for electrons. For example $E = hf$ does not apply to electrons.

Theory of knowledge (TOK)

- A useful topic for discussion in TOK is the fact that quantum physics has eliminated the clean distinction between particles and waves found in classical physics. Language is therefore unable to describe correctly these physical objects and mathematics remains the only reliable method of description.
- The role played by beauty in discovering new results is often discussed in TOK in the context of physics and mathematics. In mathematics, an often-heard quote is a saying by G. H. Hardy that 'there is no place for ugly mathematics'. In physics, perhaps the most ardent proponent of the idea of beauty as a guiding principle was one of the true greats of 20th-century physics – but also one of the least known to the general public, Paul Dirac.

Beauty and elegance – the case of Paul Dirac

In 1928, when he was just 26 years old, Dirac proposed his famous equation describing the behaviour of electrons moving at relativistic speeds. Dirac did for the relativistic electron what Schrödinger had done for the non-relativistic electron. Dirac, who was an atheist, many times said that 'God used beautiful mathematics in creating the world'. (After a lecture by Dirac on why God does not exist, Wolfgang Pauli said: 'If I understand Dirac correctly, his meaning is this: "there is no God, and Dirac is his Prophet".')

The reference to the words 'beauty' and 'elegance' appears in the work of many physicists and Dirac was perhaps the leading example of this. In a 1963 Scientific American article he said:

> It is more important to have beauty in one's equations than to have them fit experiment … It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress. If there is not complete agreement between the results of one's work and experiment, one should not allow oneself to be too discouraged, because the discrepancy may well be due to minor features that are not properly taken into account and that will get cleared up with further developments of the theory.

Indeed, Dirac's equation, derived in 1928, predicted one phenomenon that had never been observed and explained in a natural way some puzzling features of atomic spectra. These were explained when his equation showed the natural emergence of a property of the electron called spin. There was also the extraordinary prediction that there should exist a particle of the same mass but opposite charge to the electron, the anti-electron (or positron as it came to be called later). The positron was discovered in 1931.

His equation is considered to be one of the 'beautiful' equations of physics. It states that:

$$(\gamma \bullet \partial + m)\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

To close, here are two Dirac quotes:

> The mathematician plays a game in which he himself invents the rules while the physicist plays a game in which the rules are provided by nature, but as time goes on it becomes increasingly evident that the rules which the mathematician finds interesting are the same as those which nature has chosen.

> The research worker, in his efforts to express the fundamental laws of Nature in mathematical form, should strive mainly for mathematical beauty. He should take simplicity into consideration in a subordinate way to beauty … It often happens that the requirements of simplicity and beauty are the same, but where they clash, the latter must take precedence.

